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THE EFFECTIVENESS OF DISCOVERY LEARNING MODEL WITH A REALISTIC MATHEMATICS EDUCATION APPROACH IN ENHANCING STUDENT'S MATHEMATICAL REASONING SKILLS

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ABSTRACT

This study aims to determine the effectiveness of the Discovery Learning model with the RME approach on students' mathematical reasoning skills. This study employed a quasi-experimental method using a pretest-posttest control group design. The study population consisted of all eighth-grade students at a public junior high school in Tasikmalaya. The sample was selected using cluster random sampling, consisting of an experimental class that received instruction using the Discovery Learning model with the RME approach and a control class that received instruction using the PBL model. The research instrument was a mathematical reasoning ability test. Data were analyzed using descriptive statistics and inferential statistics via the Independent Samples t-Test. The results of the study indicate that the value of $0.017 < 0.05$, so mathematical reasoning skills of students who received instruction using the Discovery Learning model with the RME approach are significantly better than those of students who received instruction using the PBL model. This indicates that the mathematical reasoning ability of students who received instruction using the Discovery Learning model with the RME approach is better than that of students who received instruction using the PBL model. Thus, the Discovery Learning model with the RME approach is effective in improving students' mathematical reasoning ability.

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1. INTRODUCTION

Mathematical reasoning is one of the key competencies that students must possess in mathematics education. This ability reflects students' capacity to understand relationships between concepts, recognize patterns, make conjectures, and draw logical conclusions (Hendriana et al., 2017). The National Council of Teachers of Mathematics (NCTM, 2000) emphasizes that reasoning is a process standard in mathematics learning that students must possess to connect ideas, make generalizations, and draw logical conclusions. In line with this, Ministry of Education and Culture Regulation No. 22 of 2016 stipulates that mathematics learning aims to enable students to understand concepts, recognize patterns, solve problems, and present arguments effectively (Husniah & Azka, 2022). Thus, mathematical reasoning ability is one of the key competencies that needs to be developed in mathematics education (Nurmilah et al., 2023).

However, various findings in the field indicate that students' mathematical reasoning skills remain relatively low. Amin et al. (2023) found that at one junior high school in Cirebon Regency, only about 20% of students were able to solve reasoning problems, while the majority could only answer questions at a low level of understanding. Sari et al. (2021) also found that students struggled to present mathematical statements both orally and in writing, were unable to draw conclusions, and could not provide reasons or evidence for the solutions they offered. A similar situation was observed at SMPN 8 Tasikmalaya based on pre-research interviews with eighth-grade mathematics teachers, where most students still faced challenges in interpreting word problems, formulating possible solutions, and providing logical justifications for the correctness of their answers all of which align with the indicators of mathematical reasoning ability. This situation is closely tied to the instructional model implemented in the classroom. Although teachers have employed the Problem-Based Learning (PBL) model, the steps within that model have not provided students with sufficiently specific opportunities to construct evidence and formulate arguments independently, thereby failing to fully facilitate students' mathematical reasoning skills.

Various efforts at educational innovation have been made to address the low levels of students' mathematical reasoning. Prior research has extensively explored diverse instructional methodologies to mitigate this issue. For instance, Mendrofa & Mendrofa (2022) demonstrated that the Discovery Learning model combined with a scientific approach yields a significant positive impact on the mathematical reasoning skills of eighth-grade students, resulting in a moderate average N-Gain of 0.55. From a contextual perspective, Amelia et al. (2025) indicated that the standalone application of the Realistic Mathematics Education (RME) approach significantly enhances students' mathematical reasoning skills through structured situational problem-solving. Furthermore, the integration of discovery-based frameworks with RME has been attempted in other cognitive domains, Ashari et al. (2023) established that combining Discovery Learning with RME effectively enhances students' mathematical creative thinking skills.

Despite these valuable insights, a critical empirical gap remains in the existing literature. While Discovery Learning has been paired with generic scientific approaches for reasoning, and its combination with RME has been largely restricted to divergent cognitive dimensions like creative thinking, empirical evidence regarding the synergetic effect of an integrated Discovery Learning-RME framework specifically targeting mathematical reasoning is virtually non-existent. Previous studies often treat discovery processes and contextual mathematization as parallel, isolated systems rather than interlocking mechanisms. Consequently, there is an incomplete understanding of how the inductive, self-directed stages of discovery can be structured by the progressive formalization inherent in

RME to support rigorous mathematical proofs and inferences. The novelty of this study lies in its systematic operationalization of this integrated model to explicitly target the core tenets of mathematical reasoning, mapping the syntax of Discovery Learning such as data collection and verification directly onto the horizontal and vertical mathematization processes of RME to provide a rigorous pathway for students to formulate conjectures and validate logical proofs.

The Discovery Learning model, combined with the RME approach, was selected as a relevant solution. The integration of the Discovery Learning model with the RME approach is seen as having the potential to enhance students' mathematical reasoning skills. The Discovery Learning model encourages students to discover concepts independently through a process of observation, exploration, and systematic inquiry (Syamsidan et al., 2023). Meanwhile, the RME approach helps students relate mathematical concepts to real-life situations, making learning more meaningful (Juandi et al., 2022). Through the integration of these two approaches, students are expected not only to be able to discover concepts independently but also to develop the ability to identify patterns, formulate hypotheses, provide logical reasoning, and draw conclusions in solving mathematical problems.

Based on the above discussion, this study aims to determine the effectiveness of the Discovery Learning model with the Realistic Mathematics Education approach in improving students' mathematical reasoning skills.

2. METHOD

This study employed a quasi-experimental design, specifically a pretest-posttest control group design. This design involved two groups: an experimental class that received instruction using the Discovery Learning model with a Realistic Mathematics Education (RME) approach, and a control class that received instruction using the Problem-Based Learning (PBL) model. The research design is presented in table 1.

Tabel 1 Research Design

Group	Experiments	Treatment	Control
Exsperimen	O_1	X_1	O_2
Control	O_3	X_2	O_4

Note:
X_1 = Discovery Learning with RME approach
X_2 = Problem Based Learning (PBL)

The study population consisted of all eighth-grade students at SMPN 8 Tasikmalaya, comprising 11 classes. The sampling technique used was cluster random sampling, a method of determining the sample by using groups within the population as the basis for sample selection (Creswell, 2023). Based on this technique, eighth-grade class J was selected as the experimental class consisting of thirty-five students, and eighth-grade class K was selected as the control class consisting of thirty-two students.

The research instrument consisted of a mathematical reasoning test in the form of 1 open-ended essay question which comprised points (a), (b), (c), and (d). These sub-questions were developed based on four main indicators of mathematical reasoning skills adapted from Hendriana et al. (2017): (1) formulating conjectures, (2) performing mathematical manipulations, (3) providing evidence and reasoning for solutions, and (4) drawing logical conclusions.

The research procedure was carried out in three systematic phases. First, the preparation phase involved analyzing the curriculum, developing Lesson Plans (LP), creating Student Worksheets (SW) based on Discovery Learning and RME, and designing the test instruments. Second, the implementation phase began with administering a pretest to both classes to measure their initial abilities. This was followed by providing treatments for 5 sessions, where the experimental class was taught using Discovery Learning with the RME approach, while the control class was taught using the PBL model. Third, the evaluation phase concluded the treatment by administering a posttest using the same test instrument to measure the improvement in students' mathematical reasoning skills.

Before implementation, the test instrument was validated by one expert in mathematics education to evaluate its content validity. The validation process included assessing the alignment of test items with indicators of mathematical reasoning ability, as well as the clarity of language and accuracy of content. Subsequently, the instrument was pilot-tested on students outside the study population to assess its validity and reliability.

Data collection was conducted via a pretest and a posttest. The data analyzed were the Normalized Gain (N-Gain) scores, calculated using the formula proposed by Sukarelawan et al. (2024) as follows:

$$N - Gain = \frac{Posttest\ Score - Pretest\ Score}{Ideal\ Score - Pretest\ Score}$$

Statistical analysis was performed using descriptive and inferential statistics. The prerequisites for data analysis included normality testing via the Shapiro-Wilk test and homogeneity testing via Levene’s Test. After both prerequisite tests were met, an Independent Sample T-Test (One-Tailed) was conducted to verify the research hypotheses. The statistical hypotheses tested in this study are formulated as follows:

$$H_0 = \mu_1 \leq \mu_2$$
$$H_1 = \mu_1 > \mu_2$$

Where:
μ_1: is the mean parameter of the experimental group
μ_2: is the mean parameter of the control group

The hypothesis testing was carried out at a significance level of $\alpha = 0{,}05$. The decision-making criteria referred to Field (2005) dictated that if $\frac{1}{2} \times sig.(2 - tailed) < 0{,}05$, then H_0 is rejected, meaning that the experimental class achieved a significantly better improvement than the control class.

3. RESULTS AND DISCUSSION

3.1. Results

Before administering the test instrument to collect the primary research data, the instrument underwent a two-stage validation process to ensure its quality and appropriateness. First, a content validity assessment was conducted by expert validators (expert judgment). The evaluation from the expert lecturers indicated that the 1-item essay test which comprises sub-items 1a, 1b, 1c, and 1d was qualitatively appropriate, feasible, and ready for classroom implementation without structural modifications. Second, an empirical instrument trial was conducted involving 40 students to evaluate its statistical validity and reliability. The empirical validity test was analyzed using the product-moment correlation framework. The analysis revealed that all sub-items yielded correlation coefficients higher than the critical t-table value ($t_{table} = 0{,}68118$), thereby declaring all items statistically valid. Furthermore, the reliability analysis yielded a Cronbach's Alpha coefficient of 0.806, which exceeds the critical r-table value ($r_{table} = 0{,}325$). This indicates a very high degree of internal consistency, confirming that the instrument is highly reliable and dependable for measuring students' mathematical reasoning skills. The detailed statistical results of the instrument validity trial are summarized in table 1.

Table 1 Validity Results

Test Item	Correlation Coefficient	t_{table}	Decision
1a	0,793	0,68118	Valid
1b	0,866	0,68118	Valid
1c	0,708	0,68118	Valid
1d	0,818	0,68118	Valid

Following the instrument verification, the primary data analysis commenced with a descriptive analysis to evaluate the students' performance, followed by an inferential analysis to test the formulated hypotheses. The descriptive analysis results show that students' mathematical reasoning skills were evaluated based on four main indicators: formulating conjectures, performing mathematical manipulations, providing evidence and justifications for solutions, and drawing conclusions. Specifically, a comparison of the percentage of achievement for each indicator in the experimental and control classes before and after the intervention is presented in table 2.

Tabel 2 Percentage of Achievement of Mathematical Reasoning Skills Indicators

Class	Indicator	Pretest (%)	Posttest (%)	Improvement (%)
Experimental	Formulating Conjectures	54,55	77,38	22,83
	Performing Mathematical Manipulations	24,24	91,67	67,43
	Providing Evidence and Reasoning for Solutions	5,05	64,29	59,24
	Drawing Conclusions	9,09	53,57	44,48
Control	Formulating Conjectures	45,98	77,01	31,03
	Performing Mathematical Manipulations	10,34	75,86	65,52
	Providing Evidence and Reasoning for Solutions	6,90	47,13	40,23
	Drawing Conclusions	10,34	50,57	40,23

Table 2 shows an improvement in all indicators in both classes following the implementation of the learning process. In general, the achievement of indicators in the experimental class was higher than in the control class, particularly for the indicator of performing mathematical manipulations, which reached 91.67% in the experimental class. Additionally, the N-Gain data show that the average N-Gain score for the experimental class was 0.6329 and for the control class was 0.5372; both fall into the moderate category, as presented in Table 3 below.

Tabel 3 N-Gain Score Calculation Results

Class	N	$\bar{x}$	σ	σ^2
Experimental	28	0,6329	0,1912	0,037
Control	28	0,5372	0,1545	0,027

The frequency distribution of N-Gain by category is presented in Table 4.

Tabel 4 Frequency Distribution of N-Gain by Category

Class	Category	Interval	Frequency
Experimental	High	$0{,}70 \leq g < 1{,}00$	13
	Moderate	$0{,}30 \leq g < 0{,}70$	15
	Low	$0{,}00 < g < 0{,}30$	0
Control	High	$0{,}70 \leq g < 1{,}00$	5
	Moderate	$0{,}30 \leq g < 0{,}70$	20
	Low	$0{,}00 < g < 0{,}30$	3

Following the descriptive analysis, the data analysis continued with prerequisite tests and hypothesis testing. The prerequisite tests included tests of normality and homogeneity. The results of the normality test using the Shapiro-Wilk test are presented in table 5.

Tabel 5 Normality Test Results

Group	Sig.	α	Decision
Experimental	0,261	0,05	H_0 accepted
Control	0,075	0,05	H_0 accepted

Based on Table 5, the p-values for both groups are greater than 0.05; therefore, H_0 is accepted, and it can be concluded that the N-Gain data for both groups are normally distributed. Furthermore, the results of the homogeneity test using Levene's Test are presented in Table 6.

Tabel 6 Homogeneity Test Results

Sig.	α	Decision
0,327	0,05	H_0 accepted

Based on Table 6, the p-value of 0.327 is greater than 0.05; therefore, H_0 is accepted, and it can be concluded that the two groups have homogeneous variances. Since both prerequisite assumptions are met, the hypothesis testing proceeds with a one-tailed independent samples t-test. The test results are presented in Table 6.

Tabel 7 Independent Sample T-Test Results (One-Tailed)

Sig. (2-tailed)	$\frac{1}{2}$ Sig.	α	Decision
0,034	0,017	0,05	H_0 rejected

Based on Table 7, the value of ½ sig. is $0.017 < 0.05$, so H_0 is rejected. These results indicate that the mean N-Gain for the experimental class (0.6329) is greater than that of the control class (0.5372). Thus, it can be concluded that the Discovery Learning model with the Realistic Mathematics Education (RME) approach is more effective than the Problem-Based Learning (PBL) model in improving students' mathematical reasoning skills.

3.2. Discussion

The results of the study indicate that the implementation of the Discovery Learning model using the Realistic Mathematics Education (RME) approach has a positive effect on improving students' mathematical reasoning skills. This finding is evidenced by the prominent enhancement in the experimental class's reasoning achievements across all evaluated parameters compared to the control group, with the most notable progress observed in students' capacity to manipulate mathematical constructs structurally. These findings indicate that the integration of the Discovery Learning model with the Realistic Mathematics Education (RME) approach is effective in encouraging students to formulate conjectures, perform mathematical manipulations, prove solutions, and draw logical conclusions in mathematics learning.

In the indicator formulating hypotheses, learning that begins with contextual problems through the stimulation stage helps students understand the problem situation before formulating hypotheses. Gradually, students are able to connect real-world problems regarding measures of central tendency with their prior knowledge, enabling them to formulate potential solutions in a more focused manner. This aligns with Freudenthal's view, which emphasizes that mathematics becomes more meaningful when learning begins with contexts close to students' lives, allowing them to construct mathematical concepts themselves through a gradual process of mathematization (Gravemijer, 1994).

In the indicator for performing mathematical manipulations, the characteristic of the use of models which is integrated into the data collection and data processing stages plays a crucial role in helping students transform real-world problems into mathematical representations. Students use simple models in the form of tables as a bridge in the process of moving from horizontal to vertical mathematization. This aligns with Gravemijer (1994) explanation that models in RME function as a bridge that helps students transition from understanding concrete situations to more formal mathematical concepts.

Regarding the indicator that requires students to provide evidence and reasoning for their solutions which was the most challenging indicator for students to master in this study the interactive elements integrated into the verification stage provided students with opportunities to validate their hypotheses through group discussions and class presentations. Through these activities, which were facilitated repeatedly at each meeting, students gradually began to get used to communicating and justifying the problem-solving process they had carried out. This aligns with the research by Rahmawati & Astuti (2022), which suggests that indicators of mathematical reasoning ability particularly the formulation of proofs and the provision of justifications for the validity of solutions can develop more optimally through continuous interactive and collaborative learning.

In the "drawing conclusions" indicator, the generalization stage integrated with the characteristics of "the use of students' own productions and constructions" and "intertwinement" provides students with the opportunity to formulate conclusions based on an independent exploration process. This aligns with Baroody's (Hendriana et al., 2017) assertion that strong mathematical reasoning skills enable students not merely to memorize facts and procedural steps, but to use them meaningfully to draw logical and relevant conclusions.

In addition to improving mathematical reasoning skills, the implementation of the Discovery Learning model with the RME approach also contributes to increasing students' active engagement in the learning process. Students demonstrate more active participation because the problems presented are directly related to their real-life experiences. This aligns with the research by Ashari et al. (2023), which shows that the Discovery Learning model with the RME approach can improve students' mathematical thinking skills by providing a space for exploration directly connected to real-life situations. Furthermore, research by Amelia et al. (2025) also indicates that the RME approach significantly enhances students' mathematical reasoning skills by fostering meaningful mathematization processes rooted in real-life contexts.

Improvements in each indicator suggest that integrating the Discovery Learning model with the RME approach provides students with opportunities to be more active in exploring ideas, constructing knowledge independently, and connecting mathematical concepts to real-life situations. Additionally, students become more engaged in the learning process because the problems presented are relevant to their daily experiences. These findings carry substantial practical implications for mathematics education, suggesting that shifting toward a hybrid framework of guided discovery and contextual realism can drastically reinforce students' analytical logic and provide teachers with an effective strategy to anchor concepts through target-specific lesson materials. However, this study has limitations, including a sample that is relatively limited to a single school, which may affect the generalizability of the findings. Therefore, for future research, it is recommended to involve a larger and more diverse sample and to extend the duration of the intervention to obtain more comprehensive and generalizable results.

4. CONCLUSION

Based on the results of the analysis and discussion, it can be concluded that the research problem regarding students' low mathematical reasoning skills prior to the implementation of the learning model was successfully addressed through the application of the Discovery Learning model using the Realistic Mathematics Education (RME) approach. The implementation of this integrated model proved to be more effective than the Problem-Based Learning (PBL) model in improving students' mathematical reasoning skills. This improvement is comprehensively observed across all evaluated reasoning indicators, which include formulating conjectures, performing mathematical manipulations, providing evidence and justifications for solutions, and drawing conclusions.

These findings imply that the Discovery Learning model with the RME approach can be considered as a strategic and alternative instructional framework for mathematics teachers to systematically build students' analytical logic and conceptual understanding in the classroom. By combining self-directed discovery and realistic contexts, the learning process can guide students to transition naturally from understanding concrete situations to formal mathematical structures. However, this study was limited to a relatively small sample size drawn from a single school and specific material, suggesting the need for further research involving a larger, more diverse student population and an extended intervention duration to obtain more comprehensive and widely generalizable results.

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